Exponential Growth

The value of invested money grows or shrinks EXPONENTIALLY.

This means that if you have invest $1000 at a rate of 10%:

In the first year, $100 will be added and the value will grow from $1000 to $1100.

In the 2nd year, $110 will be added and the value will grow from $1100 to $1210.

In the 3rd year, $121 will be added and the value will grow from $1210 to $1331.

Etc.

There is an equation the we can use to model this growth, so that we do not need to continue to add these numbers.

Compound Interest :

$$V = p(1+r)^t$$

V is the Value.

p is the principal or original amount invested.

r is the rate of growth (percent as a decimal - this can be negative.)

t is the time - usually in years if we are computing annual growth.

Use the equation above and a calculator to determine what happens to the value of money invested at the given rates and times:

1. $1000 invested at 3% after 10 years.

   _____

2. $300 invested at 10% after 10 years.

   _____

3. $500 invested at 7% after 10 years.

   _____

4. $1000 invested at 3% after 20 years.

   _____

5. $300 invested at 10% after 20 years.

   _____

6. $500 invested at 7% after 20 years.

   _____

Which investment is worth the most after 10 years? 20? Why?

_____

_____

Exponential Growth

Application:

To pay for college, three funds were started to cover your expenses. You now have the option to choose which of the three funds you would like to keep to help you pay for college. Explain which you would choose and why:

Option 1:

$5000 was placed into a checking account (no interest) at your birth. After that, $1000 a year is placed into the account until you are 18. How much will the account be worth at each age below?

Age 5: _____   Age 10: _____   Age 15: _____   Age 18: _____

Option 2:

$9000 is invested at the time of your birth in government bonds, earning an average of 4% per year.

Age 5: _____   Age 10: _____   Age 15: _____   Age 18: _____

Option 3:

$3500 is invested at the time of your birth into a market account which can gain or lose money, but in the long term averages 11% growth.

Age 5: _____   Age 10: _____   Age 15: _____   Age 18: _____

Which is your best option? Which is your parents' best option? Why?

_____

_____

Practice:

Write an equation for each of the following investments. Graph both equations on a graphing calculator to answer the following questions.   note: y is value, x is time.

Investment A:

Principal: $1,900 Interest: 8.5%

Investment B:

Principal: $4,100 Interest: 4%

Use the following window range: Xmin: -1 Xmax: 25 Ymin: -2,000 Ymax: 15,000

About how many years does it take investment A to be worth more than B?   _____

Exponential Growth

For each problem, graph the equations and answer the questions that follow.

$$V = p(1 + r)^t$$

1. You have $100 invested in an account that earns 7% interest annually, and $200 in an account that earns 4% annually. Graph an equation to represent each account then answer the questions that follow. Use the window range below.

XMIN -2   XMAX 30   YMIN -50   YMAX 700

a. How much is the $100 investment worth after:

5 years _____   10 years _____   20 years _____   30 years _____

b. How much is the $200 investment worth after:

5 years _____   10 years _____   20 years _____   30 years _____

c. Use the **Calc** function and choose intersection. Choose each curve and a guess.

To the tenth, how many years does it take for the two investments to be equal in value? _____

To the cent, what is the value of both investments when they are equal? _____

2. Your grandfather owns two classic cars. His '66 Ford Mustang is valued at $18,000 and appreciates (increases in value) at a rate of about 2% per year. He also owns a '59 Ford Thunderbird worth $12,500 which currently appreciates at a rate of about 5.5% per year. Use the window range below to graph the value of each vehicle, assuming current appreciation rates continue.

XMIN -2   XMAX 30   YMIN -1000   YMAX 70000

a. How much is the Mustang worth after: (round to the nearest $100)

5 years _____   10 years _____   20 years _____   30 years _____

b. How much is the Thunderbird worth after:

5 years _____   10 years _____   20 years _____   30 years _____

c. To the tenth, how many years does it take for the two cars to be equal in value? _____

d. To the dollar, what is the value of both investments when they are equal? _____

Exponential Growth

Values can also decrease over time. This is called _depreciation._
Since the value is decreasing, the rate is negative and the equation looks like:

$$V = p(1-r)^t$$

3. Your other grandfather owns two new cars. His 2007 Honda is worth $16,000 and its value depreciates at a rate of 8% per year. His 2007 Ford is worth $21,000 and depreciates at a rate of 11% per year. Graph an equation to represent the value of each car to answer the questions that follow.

XMIN -2   XMAX 20   YMIN -1000   YMAX 22000

a. How much is the Honda worth after: (round to the nearest $100)

5 years _____   10 years _____   15 years _____   20 years _____

b. How much is the Ford worth after:

5 years _____   10 years _____   15 years _____   20 years _____

c. To the tenth, how many years does it take for the two cars to be equal in value? _____

d. To the dollar, what is the value of both investments when they are equal? _____

e. To the tenth of a year, how long does it take for the Honda's value to
cut in half (how many years will it take for the Honda to be worth only $8,000)? _____

f. To the tenth of a year, how long does it take for the Ford's value to
cut in half (how many years will it take for the Ford to be worth only $8,000)? _____

Caution: made-up statistics below.
4. College tuition rates in North Carolina have risen 3% per year for the past 25 years. National tuition rates have increased by 5% per year. The average tuition rate in North Carolina was $3,000 per semester 25 years ago when the national tuition rate was $1,900. Use your own values below and graph the tuition rates for the past 25 years to answer the questions below.

XMIN _____   XMAX _____   YMIN _____   YMAX _____

a. What is the NC semester tuition rate today? (to the dollar) _____

b. When did national tuition rates surpass those of NC rates? (to the tenth of a yr) _____

Exponential Growth

Population grows or shrinks EXPONENTIALLY.

This means that if you have a population of 1000 growing at a rate of 10%:

In the first year, 100 people will be added and the population will grow from 1000 to 1100.

In the 2nd year, 110 people will be added and the population will grow from 1100 to 1210.

In the 3rd year, 121 people will be added and the population will grow from 1210 to 1331.

Etc.

There is an equation the we can use to model this growth, so that we do not need to continue to add these numbers.

I am using the equation that is generally used for Compound Interest - which we will discussed and also involves Exponential Growth.

$$A = p(1+r)^t$$

A is the Amount you are attempting to find.

p is the principal or original amount.

r is the rate of growth (percent as a decimal - this can be negative, which we will discuss later.)

t is the time - usually in years if we are computing annual growth.

Use the equation above and a calculator to determine what happens to the population in the example above after the times given below.

Year	Population
0	1,000
5	
20	
50	
100	

The population increases very rapidly, doubling about every 7 years! After 100 years, you should have come up with a population of nearly 14 million.

Of course, very few populations will increase by 10% every year for 100 years, but the equation is useful in showing the effects of exponential growth over an extended period of time.

Practice: Graph the following to determine how many years it will take for the population of North Carolina to double if the current rate of growth does not change.

Current Population: 8.7 million   **Growth Rate:** 1.9%   **Years to double:** _____

Exponential Growth

Application:

Compare the populations of each pair of countries over the next 100 years. Assume that the rate of growth will remain constant. Fill in each chart, then graph the values on your calculator to answer the questions.

India and China are the planet's most populated nations. In 2000, China had about 1.30 billion inhabitants, while India had about 1.13 billion. China is growing at a rate of about .9%, while India is growing faster, at about 1.6%. Fill in the charts below to make predictions based on your observations.

Year	China (in billions)	India (in billions)
2000	1.30	1.13
2020		
2040		
2060		
2080		
2100		

Graph the 2 equations to represent the growth of each countries population on your calculator.
hint: xmin=-10 xmax=110 xscl=10 ymin=-1 ymax=6 yscl=1

To the nearest year, when does the population of India exceed the population of China?
(This is the point where the graphs intersect.)

_____

What is the population of China and India when they are equal?

_____

Approximately how many years does each country take to double its own population?

_____

Note: data below is from 2000
The United States and Indonesia are the 3rd and 4th most populous nations. The U.S. has 303 million people and grows at an annual rate of 1%. Indonesia Grows at a 1.3% annual rate, and has 235 million people. Determine what year Indonesia will surpass the U.S. to become the world's 3rd most populous nation.

_____

Exponential Growth: Population Algebra 10.E

1. On January 1, 2000, Podunk had a population of 851 and was increasing at a rate of 5% per year. Micropolis had a population of 1,560 and was shirinking at a rate of 1.2% per year. Graph equations for both towns to answer the questions that follow.

XMIN -2   XMAX 25   YMIN -200   YMAX 3500

a. How many people live in Podunk for each year listed?

2005 _____   2010 _____   2015 _____   2020 _____

b. How many people live in Micropolis for each year listed?

2005 _____   2010 _____   2015 _____   2020 _____

c. In what year does Podunk surpass Micropolis in population? _____

d. To the nearest year, how many years does it take for the population of Podunk to double? _____

e. To the year, how many years does it take for the population of Micropolis to halve? _____

f. During what year is the population of Podunk twice that of Micropolis? _____

2. There are currently about 10.8 million government jobs (not including school and hospital employ-ees) and government jobs are increasing at an annual rate of 0.5%. There are currently 7.7 million construction jobs in the U.S., and construction jobs are increasing at a rate of 0.9% per year.

a. How many government jobs will there be for each year listed?

2013 _____   2018 _____   2038 _____   2068 _____

b. How many construction jobs will there be for each year listed?

2013 _____   2018 _____   2038 _____   2068 _____

Graph:
XMIN _____   XMAX _____   YMIN _____   YMAX _____

c. In what year will the number of construction jobs
    surpass government jobs if current rates continue? _____

d. In what year will the total number of government
    and construction jobs combined pass 20 million? _____

Practice: Exponential Growth

Algebra 10.E

Solve using the information given. Round to the hundredth.

1. Beeville had a population of 5,750 in the year 2000 and is <u>growing</u> at a rate of 0.63% per year. Seatown had a population of 16,230 in 2000, and is <u>shrinking</u> at a rate of 0.75% per year. Complete the chart below and answer the questions that follow.

Year	Beeville	Seatown
2000	5,750	16,230
2020		
2040		
2060		
2080		
2100		

Graph the results on your calculator to help you answer the following:

2. Which country has a greater population after 70 years of growth?

2. _____

3. During what year are the populations equal?

3. _____

4. When the populations are equal, how many people live in each country?

4. _____

Answer: Solve using techniques we have learned.

5. How much money will you have in 10 years if you invest $20 at 18% interest?

5. (to the cent) _____

6. How many years will it take $100 to double if you are earning 7% interest?
(you may use an estimate, but use a calculator to be exact)

6. (to the tenth) _____

7. How many people will live in Wake County in 2050 if the current population is 786,000 and it has an annual growth rate of 4.1%?

7. (to the person)_____

8. Which interest rate will double your money about every 7 years?
5%, 10%, 15% or 20%?

8. _____

Practice Quiz: Exponential Growth   Algebra 10.E

Note:
Statistics and vehicles are all made-up for this quiz.

9. A 2008 Honda sells for 17,500 thousand dollars. A 2008 Volkswagen costs 25 thou-
    sand dollars. The Honda loses 8.5% of its value each year, while the VW *depreci-
    ates* at a rate of 10% per year. Complete the table below and answer the questions
    that follow.

Year	Honda (in dollars)	VW (in dollars)
2008	17,500	25,000
2013		
2018		
2023		
2028		
2033		

Graph the results on your calculator to help you answer the following:

10. Which car is worth more after 20 years?

10. _____

11. When are the values equal? (round to the tenth of a year)

11. _____

12. When the values are equal, how much is each car worth?

12. _____

13. After how many years does the value of the Honda cut in half?
     (use the graph for this answer, not an estimate. Round to the tenth.)

13. _____

14. After how many years does the value of the VW to cut in half?
     (use the graph for this answer, not an estimate. Round to the tenth.)

14. _____

Pledge: Write-out and sign.

Half-Life and Doubling Time

Algebra 10.E

$$V = p(1+r)^t$$

We have discovered that a value that grows exponentially continues to grow faster and faster.

A value that decays exponentially shrinks slower and slower.

One good measure of exponential growth is **the time it takes for a value to double.**

A good estimate of a value's doubling time can be found by **dividing the growth rate into 70**. The same estimate can be used for the **half-life** of a shrinking value (the time it takes to cut in half).

$$\frac{70}{\%}$$ **Approximate doubling/halving time.**

Example:
You invest $250 in an account earning 5% interest. About how long will it take for your money to double in value? How long before you have $1000?

Estimate the time it takes each value to double at the given rate. Check your guess with a calculator.

1. $40 increasing at 7% per year.

2. $200 increasing at 5% per year.

3. $90 decreasing at 2% per year.

4. How long will it take a population of 50,000 to cut in half if it is decreasing at a rate of 4% per year.

5. If you invest $100 into a market account, and after five years it is worth $200, approximately what percent has the investment grown each year.

Challenge. If you purchase a car for $8,000 and it is worth $2,000 after seven years, what is the percent of depreciation?