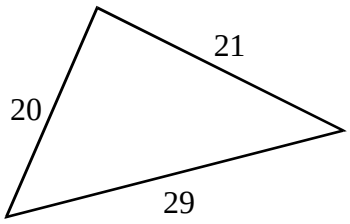
The Pythagorean Theorem

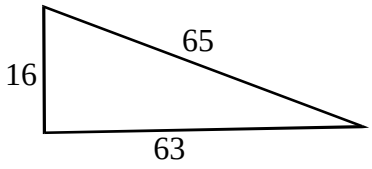
Geometry 9.2

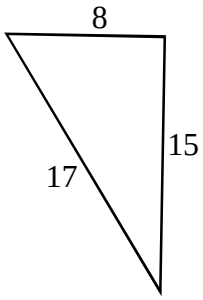
By now, you know the Pythagorean Theorem and how to use it for basic problems.

The Converse of the Pythagorean Theorem states that:
If the lengths of the sides of a triangle satisfy the Pythagorean Theorm, then the triangle is a right triangle.

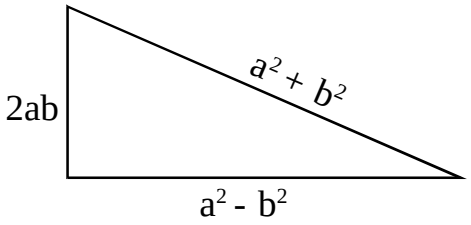
Examples:
Determine which of the following is a right triangle?







Finding all pythagorean triples:
Here is a simple method for finding every set of pythagoren triples. Given any numbers a and b where a>b:

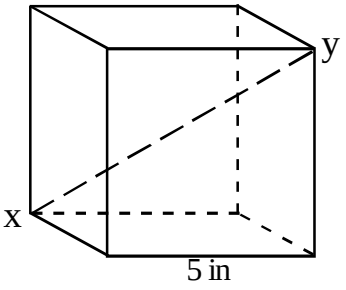


Try it!
Why does it always work?

Practice:
Using the following values for a and b, identify five pythagorean triples:

(1, 2) (1,3) (2,3) (3,4) (2,5)

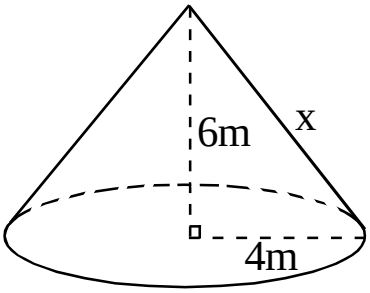
Challenge:
Determine the distance between the opposite corners of the following cube.

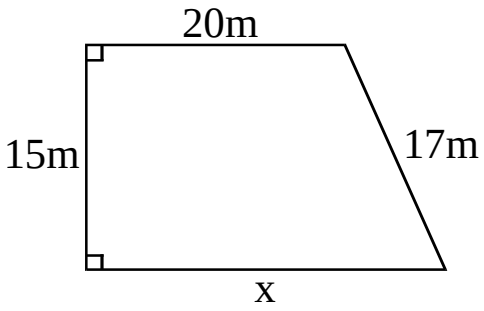


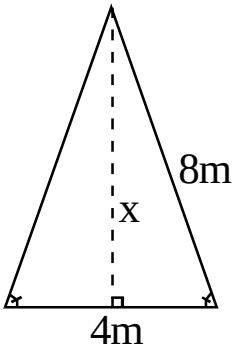
The Pythagorean Theorem

Geometry 9.2

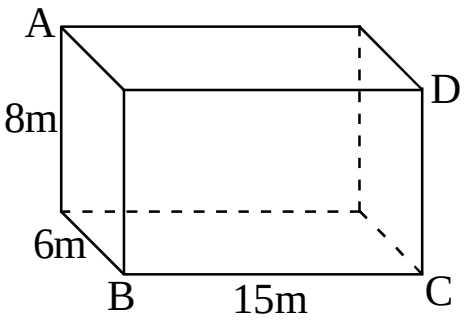
Practice:
Solve for x in each. Leave answers in simplified radical form.







Practice: Find the shortest distance between each pair of points listed for the diagram below.
Simplify Radical Answers.



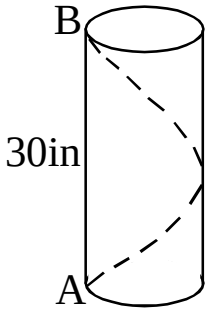
A to B: _____

A to D: _____

B to D: _____

A to C: _____

Challenge: An ant is crawling up a cylinder from point A to point B in the diagram below. Instead of climbing straight up, he climbs around the pole (in a spiral) to reach point B. How much *farther* does the ant crawl by taking a lap around the pole than if he were to just climb straight up? Round to the tenth.



Radius = 5in

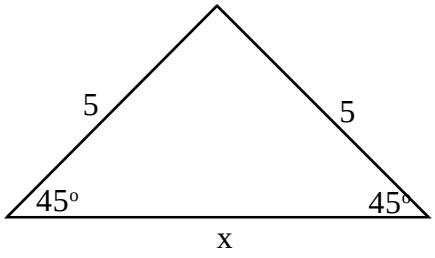
The Pythagorean Theorem

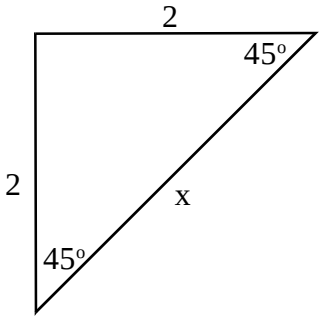
Geometry 9.3

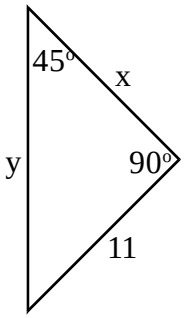
Special Right Triangles:
There are two special right triangles typicaly taught:

45-45-90

Determine the relationship between the legs and the hypotenuse in the triangles below:

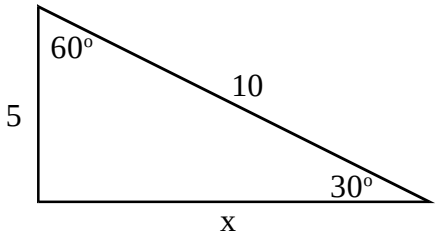


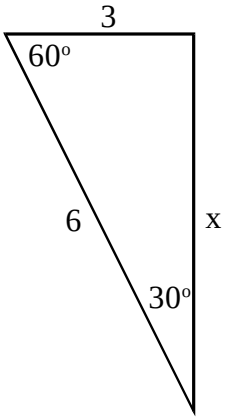


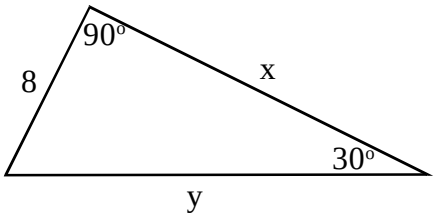


30-60-90

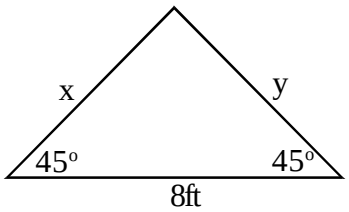
Determine the relationship between the legs and the hypotenuse in the triangles below:

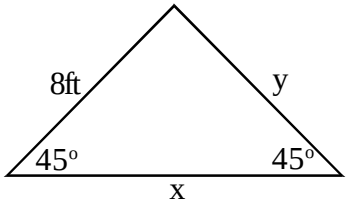


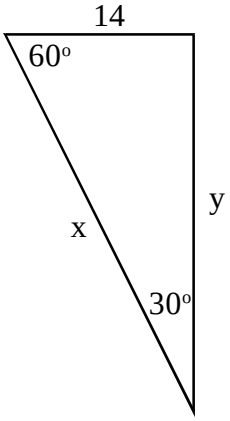


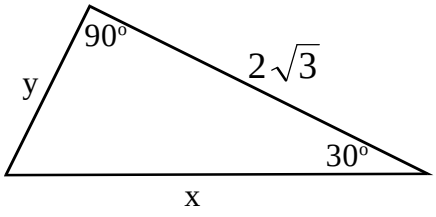


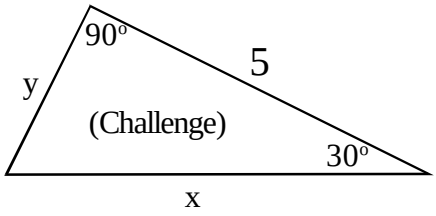
Use the rules above to find the missing sides for the triangles listed:











The Pythagorean Theorem

Geometry 9.4

Word Problems: Round to the tenth.

1. Mary and Benjamin are driving to their friend Paul's house for a birthday party. Mary drives nine miles north and six miles east to get there, while Benjamin drives three miles south and 7 miles west. How far does Mary live from Benjamin?

2. Televisions are sold based on the length diagonally across the screen. Widescreen tevevisions have an *aspect ratio* of 16:9 (the ratio of a screen's width to its height). What are the length and width of a 45" widescreen television? (remember: 45" is the diagonal length)

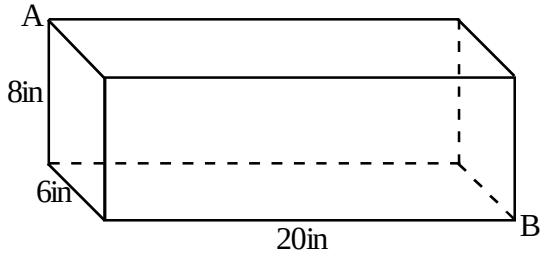
3. Michael has two ladders set up to clean the gutters of his house. The top of each ladder reaches the base of the gutter, 30 feet from the ground. One of the ladders is 32 feet tall, and the other is 33 feet tall. If the tops of the ladders are 20 feet apart, how far apart are the bottom of the ladders?

Word Problems: Round answers to the tenth.

1. Salemburg is 17 miles south of Linbrooke, and Linbrooke is 5 miles west of Pueblo. Carson lives nine miles north of Linbrooke. How many miles will Carson have to drive altogether from his home to Salemburg if he stops in Pueblo on the way?

2. Patrick is standing in the middle of a large field throwing baseballs. He throws the first ball 20 yards straight out. He turns 90 degrees to the right an throws a second ball 23 yards straight out. He turns 90 degrees to the right again and throws a third ball 45 yards (straight out again). What is the shortest distance he can walk to retrieve all three balls (he does not need to return to his original spot).

3. An ant is crawling inside of a box with the dimensions below. What is the **shortest possible distance** the ant can walk along the inside of the box to get from corner A to the food at corner B?



The Pythagorean Theorem

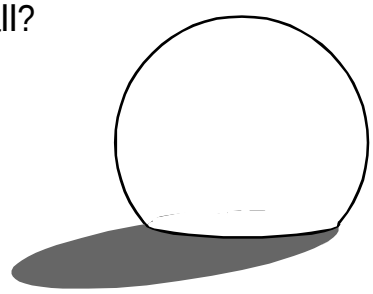
Geometry 9.4

Word Problems: Round to the tenth.

1. A fifty foot ladder rests against a wall so that the top of the ladder is 48 feet from the ground. As you start to climb the ladder, it slips and the top of the ladder drops 8 feet. How far does the bottom of the ladder slide away from the wall (from its original position)?

2. A 17-foot wire connects the top of a 28-foot pole to the top of a 20-foot pole. What is the shortest length of wire that you could use to attach the top of the short pole to the bottom of the tall pole?

3. When completely inflated, a basketball has a diameter of 10 inches. A partially deflated basketball sits on the ground so that the flat part makes a circle on the floor with a 4-inch radius. What is the height of the partially-deflated basketball?



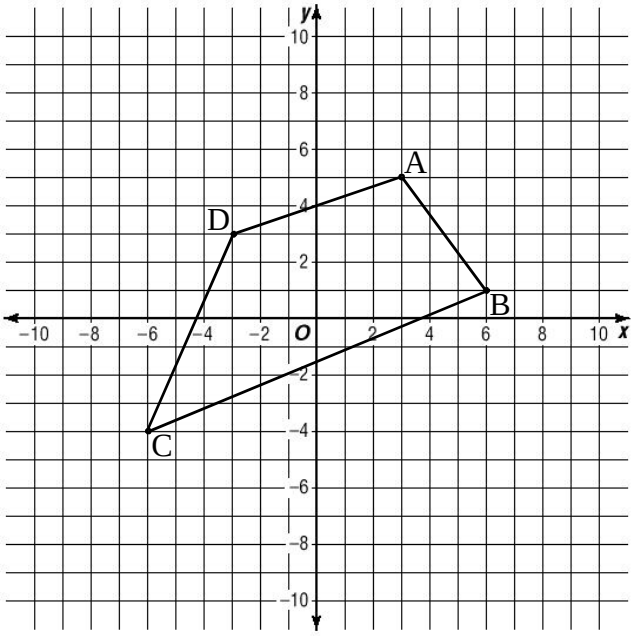
4. Riding your bicycle, you roll over an ant who squishes on the bottom of your 24-inch (diameter) bicycle tire (oops!). You roll forward and the tire makes a 1/3 (120^{o}) rotation. How high above the ground is the ant on the tire?

Distance on the Plane

Geometry 9.5

You can use the Pythagorean Theorem to find the distance between two points on the coordinate plane.

Practice:
Below is a quadrilateral drawn on the coordinate plane. Find the length of each side to determine the perimeter of the figure.



AB = _____

BC = _____

CD = _____

AD = _____

Perimeter ABCD = _____ **Challenge:** Area = _____

Of course, the distance between two points on the plane can be found without graphing:

Example:
Find the distance between the points (11,-3) and (5,-11) on the plane.

Given any two points: (x_1, y_1) and (x_2, y_2):
The distance between two points on the plane is the hypotenuse of a right triangle with a width of _____ and a height of _____.

Try to write the **distance formula** based on the pythagorean theorem:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Distance on the Plane

Geometry 9.5

The equation of a circle is very similar to the distance formula. Every point on a circle is equidistant from the center.

Given the **center of a circle (h,k)** (I do not know why (h,k) is used), and the **radius of the circle r:**

$$(x-h)^2+(y-k)^2=r^2$$

Practice:
Write an equation for a circle using the following information:

1. Center: (2,-5) Radius: 3

2. Center: (-1, 12) Radius: 7

3. Center: (9,-1) Point on Circle: (4,11)

Practice:
Name the center and radius of each circle equation below:

1. $64 = (x+3)^2 + (y-5)^2$

2. $(x-9)^2 + (y+1)^2 = 25$

3. $x^2 + y^2 = 121$

Practice:
You can use the formula to graph circles on the coordinate plane. Solve the circle equation formula for y:

$$y=\pm\sqrt{r^2-(x-h)^2}+k$$

Ex.: Begin with a circle with its center at the origin and a radius of 6:

$$y=\pm\sqrt{36-x^2}$$

Practice: Graph a circle on your graphing calculator with a radius of 6 and a center at (-2,4).

Distance on the Plane

Geometry 9.5

Practice:

Find the distance between each pair of points:

1. (3,-9) (-5, -3)

2. (-13, 5) (11, -2)

3. Find the perimeter of a triangle whose vertices are at:
A(1,3) B(-9, 27) C(9, 18) (answer in radical form)

answers: #1: 10 #2: 25 #3 26+17+(9 root 5)

Practice:

Which of the following sets of points would form a right triangle? For each, name the right angle (without graphing).

1. A(2, -1) B(5, -2) C(7, 4)

2. D(-4, -3) E(1, -1) F(-2, 4)

3. G(-10, -1) H(-6, -3) J(0, 9)

answers: #1 (right angle at B) and #3 (right angle at H)

Practice:

Write an equation for each circle described below:

1. Center: (-2, 7) Radius: 5

2. Center: (1,3) Point on the circumference: (-2, -8)

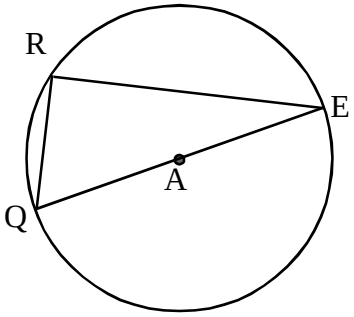
3. Endpoints of the diameter: (-9, 1) and (-1, 3)

answers: 1: $(x+2)^2 + (y-7)^2 = 25$ 2: $(x-1)^2 + (y-3)^2 = 130$ 3: $(x+5)^2 + (y-2)^2 = 17$

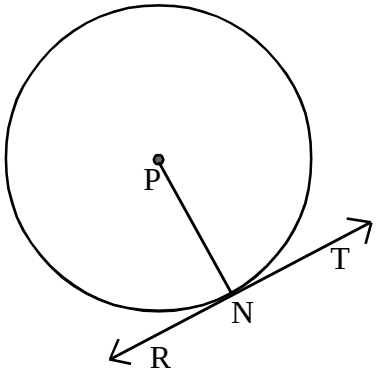
Circles and Pythagoras

Right angles in circles:

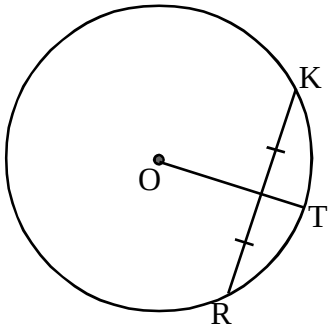
There are three obvious cases for right angles in circles:



1. Angle inscribed in a semicircle.



2. Radius and a tangent line.

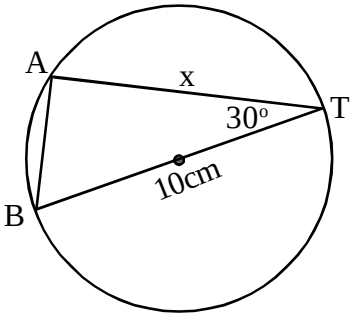


3. Radius bisects a chord.

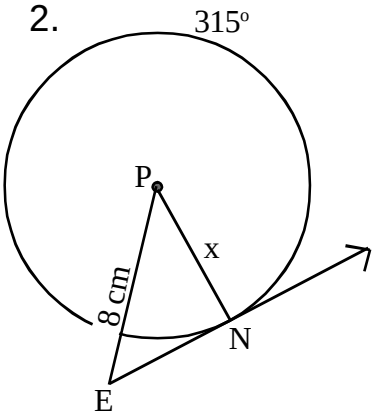
Practice:

Solve for x in each diagram below:

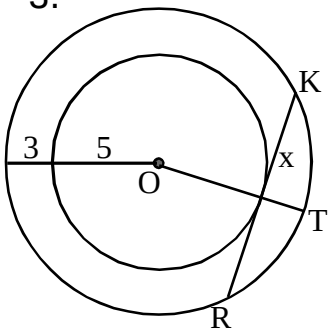
1.



2.



3.

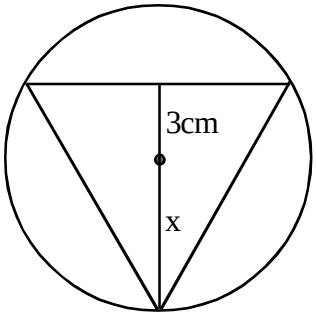


Practice:

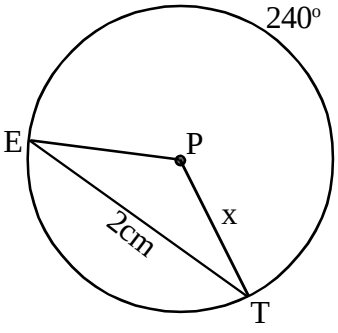
Solve for x in each diagram below:

You will need to remember your special right triangles for these.

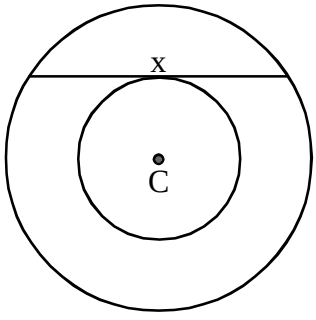
1.



2.



3.



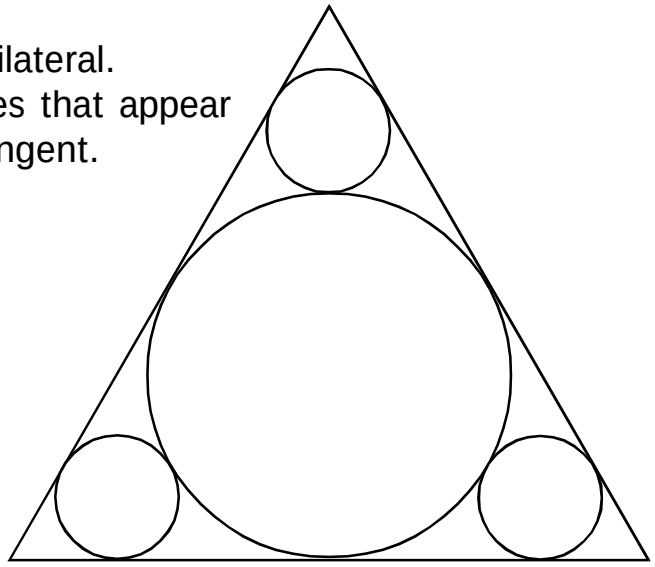
Small circle radius = 3

Large circle radius = 6

Challenge Problems

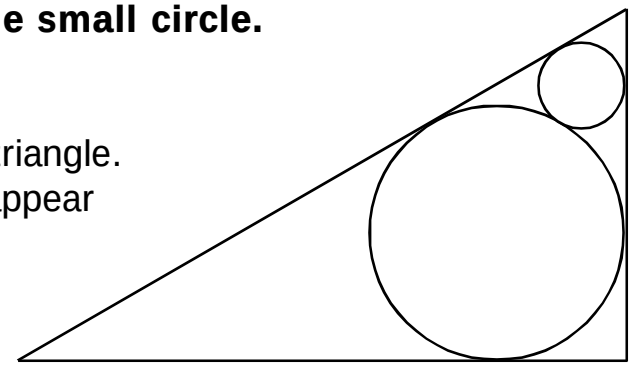
Challenge:
The radius of the large circle is 6cm.
Find the radius of the small circles.

Note:
Triangle is equilateral.
Lines and circles that appear tangent are tangent.



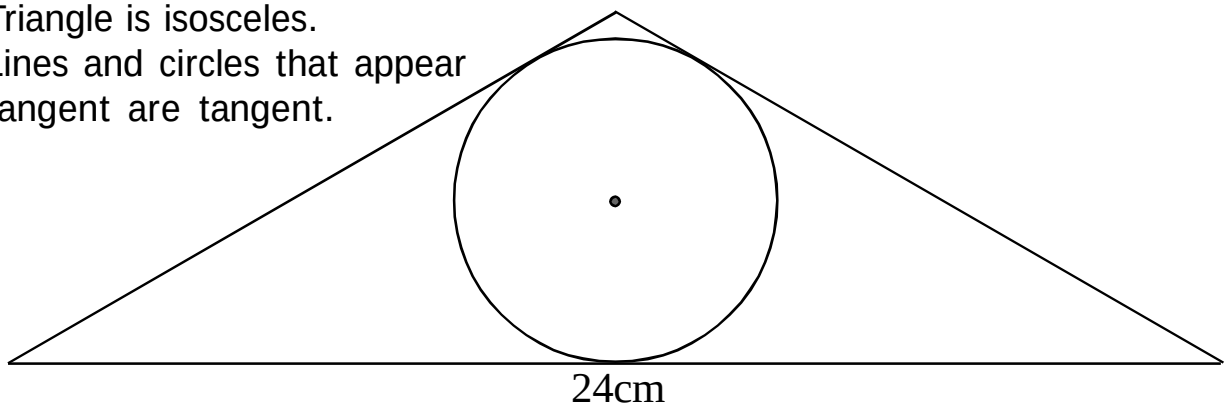
Challenge:
The radius of the large circle is 15cm.
Find the radius of the small circle.

Note:
Triangle is a 30-60-90 triangle.
Lines and circles that appear tangent are tangent.

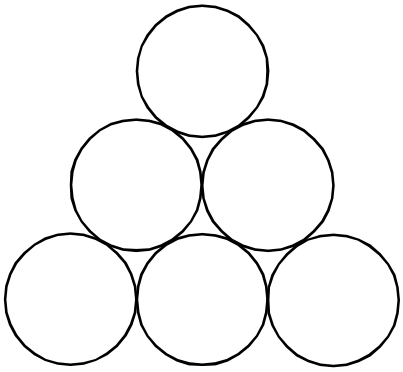


Challenge Problems

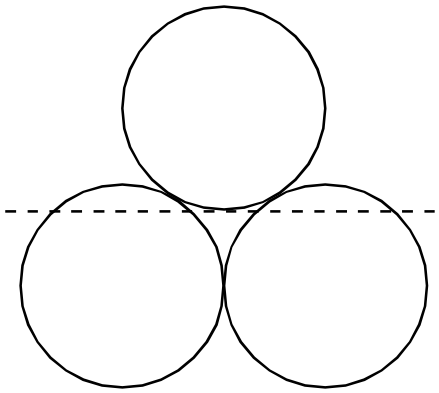
Challenge:
The vertex angle of the isosceles triangle is 120°.
Find the radius of the circle.
Note:
Triangle is isosceles.
Lines and circles that appear tangent are tangent.



Challenge:
Find the height of the stack of circles below. The circles are identical and each has a radius of 10cm.



Challenge:
Three identical cylinders are stacked as shown. What is the height of the dotted line if each cylinder has a radius of 24 inches?

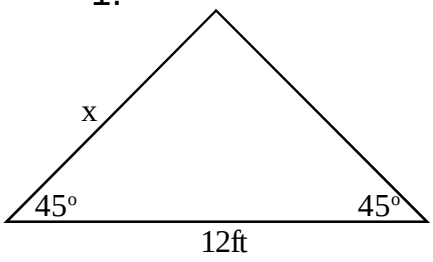


Quiz Review

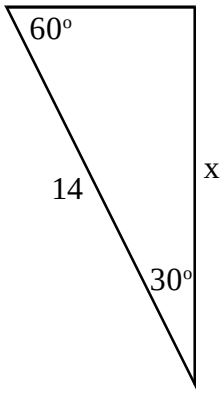
Geometry 9.6

Solve for x in each diagram below:
Simplify and rationalize radical answers.

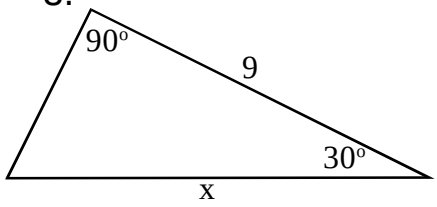
1.



2.

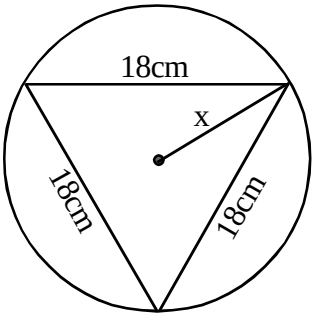


3.

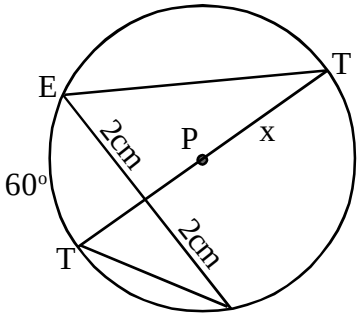


Solve for x in each diagram below:
Simplify and rationalize radical answers.

1.

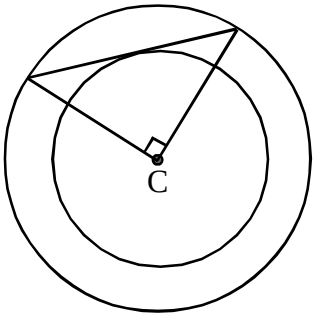


2.



x is the radius

3.



Large circle radius = 3
Small circle radius = x

Solve:

1. **In square ABCD, point A is at (-7, 3) and point C is at (0, 0). What is the perimeter of the square?**
   (hint: Find the length of AB)

2. **Colleen, Ben, and Sam start walking at noon. Each walks (at the same speed) for 60 minutes. Colleen walks north, Ben walks east, and Sam walks west. After 60 minutes, Colleen and Sam turn and walk towards each other until they meet, while Ben waits where he is. When they meet, they phone Ben, and he walks to meet them where they are. What time does Ben reach them?**

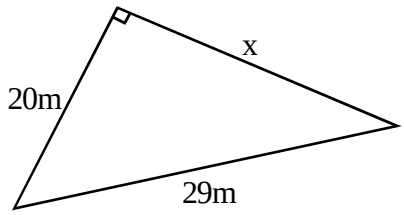
Name_____ Period _____

Practice Quiz: Pythag. Theorem

Geometry 9.6

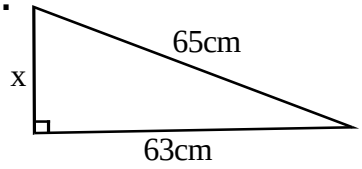
Solve for x in each diagram below: Simplify radical answers.
NO DECIMAL ANSWERS.

1.



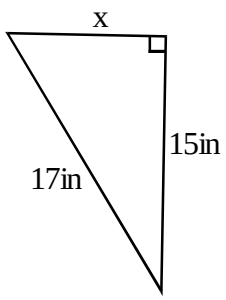
1. _____

2.



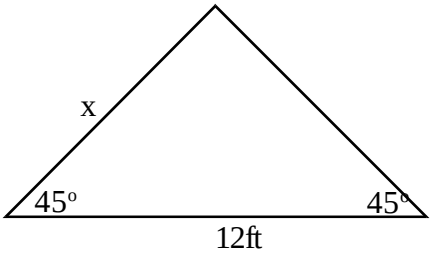
2. _____

3.



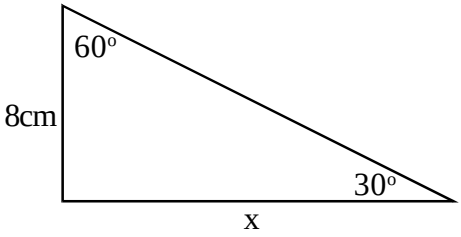
3. _____

4.



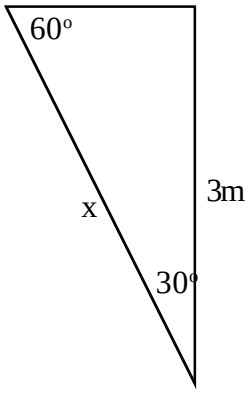
4. _____

5.



5. _____

6.



6. _____

Find the distance between each pair of points below:
Simplify radical answers.

7. (-9, 11) (3, 6)

7. _____

8. (11,-1) (4,-4)

8. _____

9. (-9, 0) (-1, 6)

9. _____

Name_____ Period _____

Practice Quiz: Pythag. Theorem

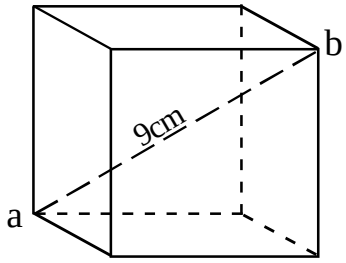
Geometry 9.6

Solve each: RADICAL FORM ANSWERS ONLY.

10. Priyanka lives two miles north of Dhruv, who lives 7 miles west of Nihar. If Paul lives one mile north of Nihar's house, how far from Priyanka does Paul live?

10. _____

11. The distance between opposite corners of a cube is 9cm. What is the **surface area** of the cube?



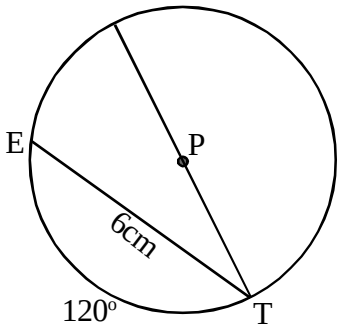
11. _____

12. The center of a circle is at (3,1) and a point on the circle is (3,-12). Which of the following points is NOT on the circle?

A: (8,13) B: (15,1) C: (-9, -4) D: (-2, -11) E: All are on the circle.

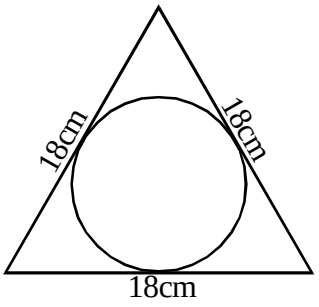
12. _____

13. What is the radius of the circle below?



13. _____

14. What is the radius of the circle below?



14. _____

Name_____ Period _____

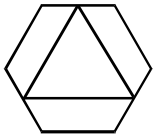
Pythagorean Area Problems

Geometry 9.8

Solve:

1. The large angles of a rhombus are twice the measure of the smaller angles. If the long diagonal is $8\sqrt{3}$ centimeters, what is the area of the rhombus?

2. What is the area of the triangle formed by connecting the midpoints of alternate sides of a regular hexagon whose sides are 12 inches long?



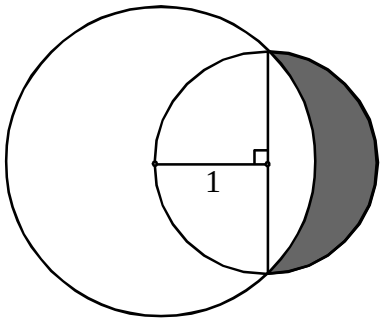
3. What is the area of the largest equilateral triangle that can fit inside a cube with an edge length of 10cm?

4. The hypotenuse of an isosceles right triangle is x. What is the area of the triangle?

5. Regular hexagon ABCDEF has sides measuring 6cm. What is the area of triangle ACE?

6. A semicircle has radius r. What is the area of the largest triangle that can be inscribed in the semicircle?

7. Find the area of the shaded lune below.

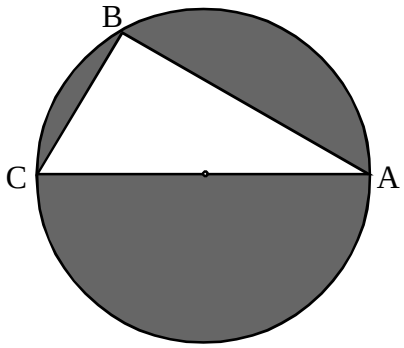


Name_____ Period _____

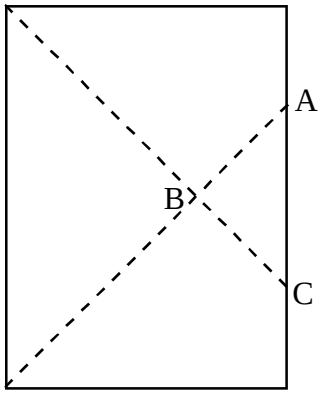
Pythagorean Problems

Geometry 9.8

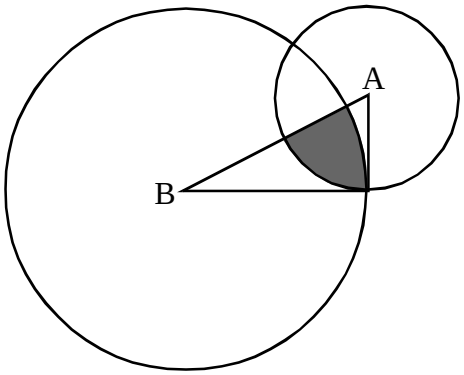
1. Find the shaded area of the figure below if angle A measures 30° and the circle radius is 4cm.



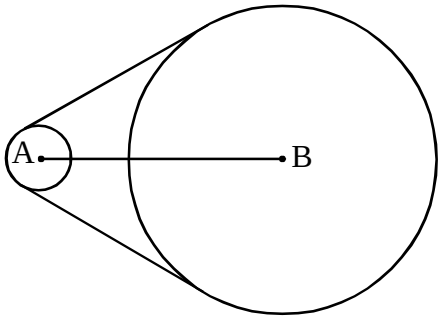
2. The top right corner of a 8.5 x 11 sheet of paper is folded down and left to align with the left edge, and the bottom right corner is folded up and left so that the fold lines look like the diagram below. What is the area of triangle abc formed by the right edge and the fold lines?



3. Circle A has a radius of 4cm. The distance and AB = 8cm. Find the area where the triangle and both circles overlap.



4. Circle A has a radius of 1cm.
Circle B has a radius of 4cm. AB=6cm.
A pulley is wrapped tightly around the two circles. How long is the pulley?



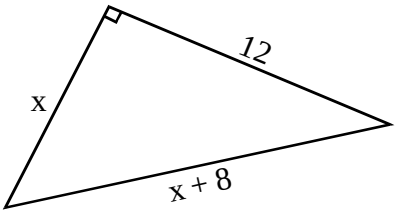
Name_____ Period _____

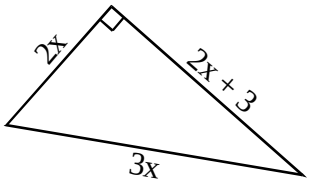
Algebra Pythagorean Problems

Geometry 9.8

Examples: Solve for x.

The following problems require the use of variables and some basic Algebra.



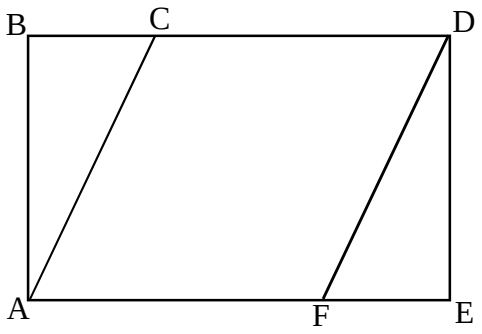


Solve:

Use similar techniques. You supply the variables.

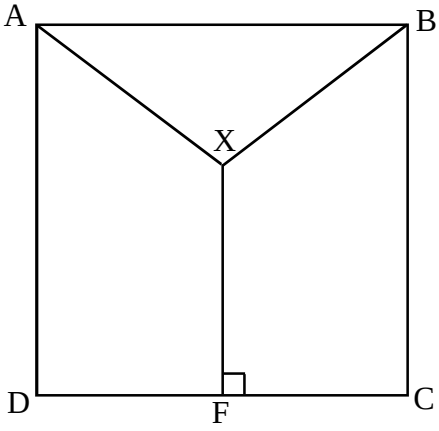
1. In rectangle ABDE:
AB = 12 and BD = 16.

Find the length of AC if ACDF is a rhombus.



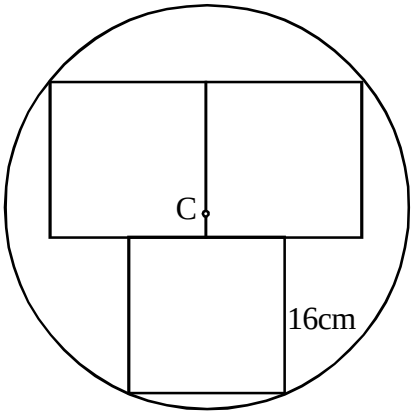
2. ABCD is a square with a side length of 24.

If AX=BX=FX, find the length of FX.



3. Circle C circumscribes three congruent squares.

Determine the area of the circle (in terms of Pi).



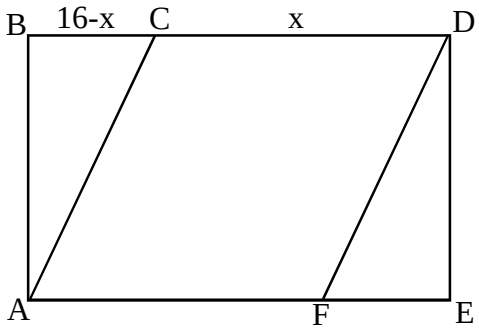
Name_____ Period _____

Hints:

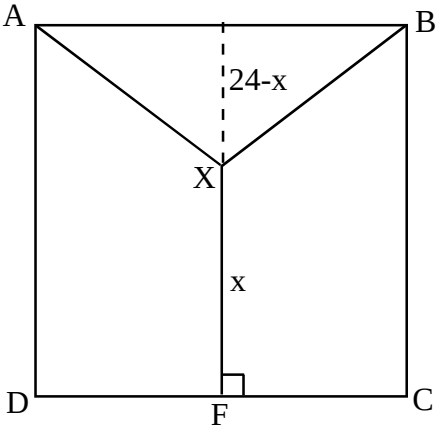
Geometry 9.8

Use only after trying on your own... you may find a different way to solve each.

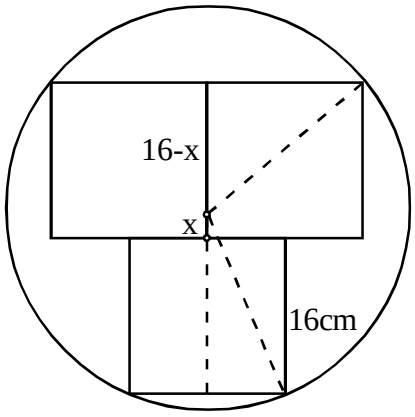
1. Solve for x using the Pythagorean Theorem.



2. Solve for x using the Pythagorean Theorem.



3. This one is trickier... Use the two right triangles to write an equation. You know the hypotenuse of each triangle is equal.



Need more?

$(16-x)^2+16^2=$_____

Name_____ Period _____

Algebra Pythagorean Problems

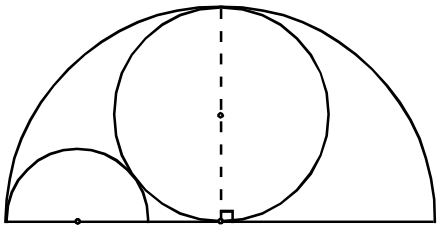
Geometry 9.8

Solve:

The following problems require the use of variables and some basic Algebra.

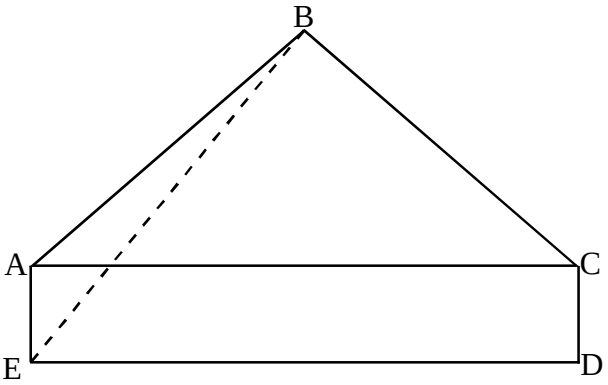
1. The radius of the large semicircle is 2cm.

Find the radius of the small semicircle.



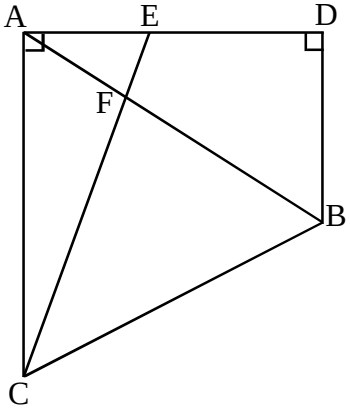
2. ACDE is a rectangle.
AB = BC = 25cm
BE = 29 cm
AE = 6 cm

Find AC.



3. AB=AC in isosceles triangle ABC.
ED=DB=2cm
CE=4cm

Find AE.



Name_____ Period _____

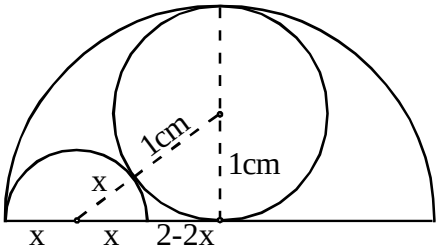
Hints

Geometry 9.8

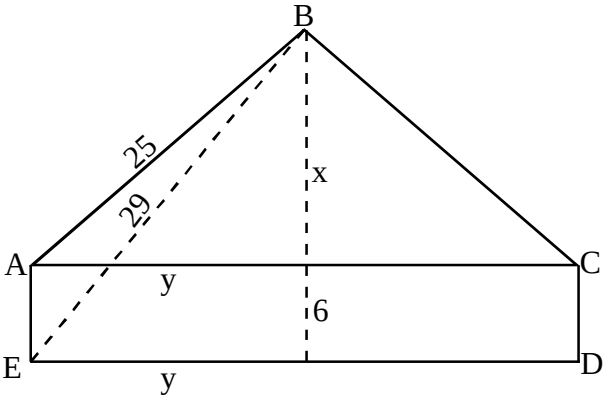
Solve:

Do not look at these hints until you have attempted each problem on your own.

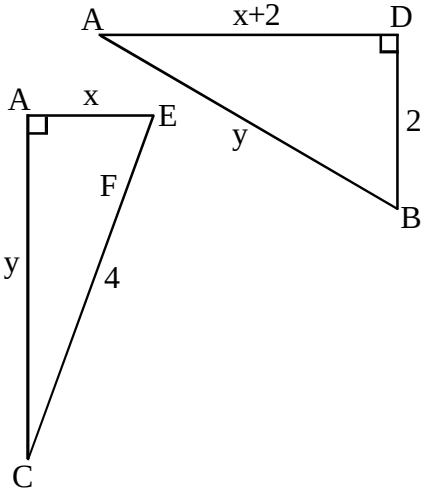
1. Use the Pythagorean Theorem to solve for x.



2. Find two expressions which are equal to y^2. Set them equal and solve for x. Use x to find y.



3. Find two expressions which are equal to y^2. Set them equal and solve for x. You will need to use the Quadratic Formula to find x.



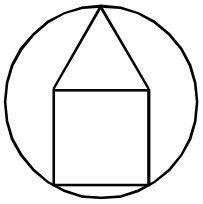
Name_____ Period _____

Really Hard Problems

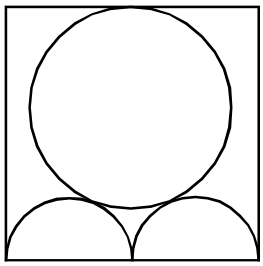
Geometry

Solve:

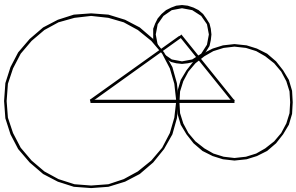
1. An equilateral triangle of side length 2cm shares a side with a square. What is the radius of the circumscribed circle?



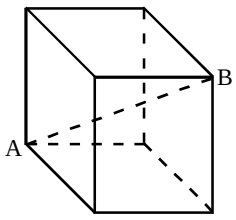
2. Two semicircles of radius 2cm have their diameters on the side of a square with a side length of 8cm. What is the radius of the large circle, which is tangent to the semicircles and the square.



3. Three circles are tangent. When their centers are connected a 3-4-5 triangle is formed. Find the combined area of the three circles.



4. The areas of the faces of the rectangular prism are 30, 40, and 50cm^2. What is the length of diagonal AB in the prism?

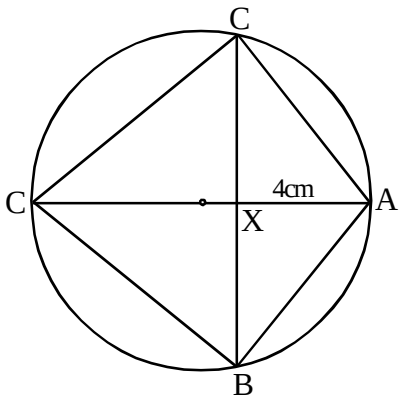


Pythagorean Problems

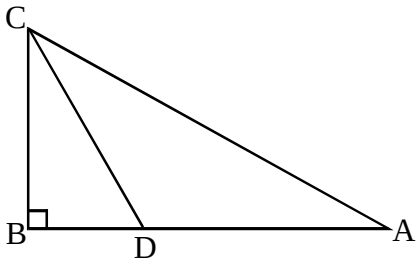
Geometry

Solve:

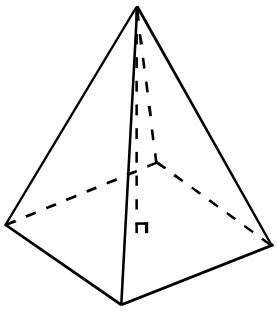
1. The diameter of the circle is 10cm and AX = 4cm. Find the perimeter of kite ABCD. Express your answers in simplest radical form.



2. In the right triangle below, angle BDC = 60°, angle BAC = 30°, and AD = 8cm. What is the area of the triangle.



3. In a right square pyramid, all of the faces have the same area. If the height of the pyramid is 3cm, what is the total surface area of the pyramid?



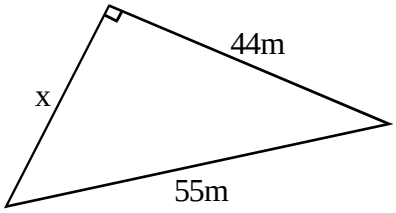
Name_____ Period _____

Practice Test: Pythagorean

Geometry 9.8

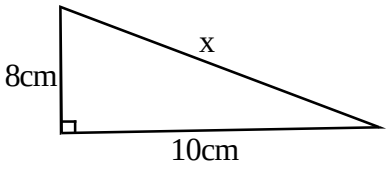
Solve for x in each diagram below: Simplify radical answers.
NO DECIMAL ANSWERS.

1.



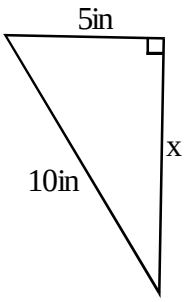
1. _____

2.



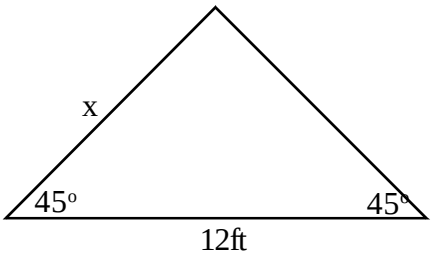
2. _____

3.



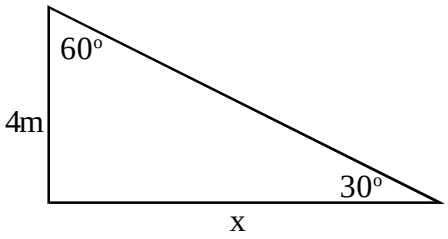
3. _____

4.



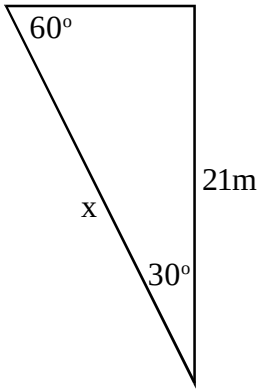
4. _____

5.



5. _____

6.



6. _____

7. What is the diagonal length of square ABCD if point A is at (2,-3) and point B is at (6, -7)?

7. _____

8. Is triangle ABC isosceles, equilateral, or scalene for the points given?
A: (2,5) B: (-1,9) C: (-4, 4)

8. _____

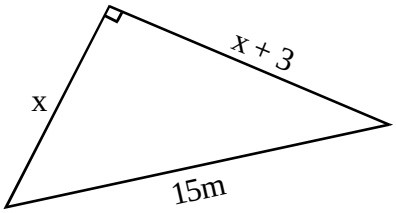
Name_____ Period _____

Practice Test: Pythagorean

Geometry 9.8

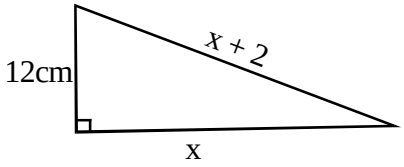
Solve for x in each diagram below: Simplify radical answers.
NO DECIMAL ANSWERS.

9.



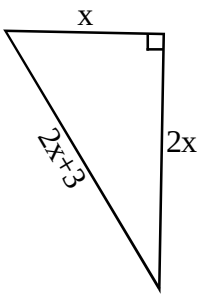
9. _____

10.



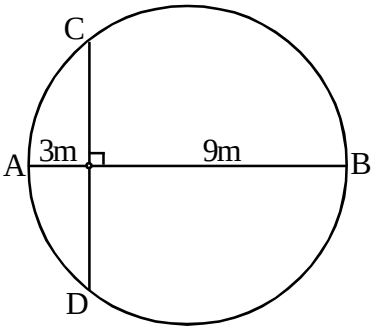
10. _____

11.



11. _____

12. In the figure below, AB is a diameter. What is the length of CD?

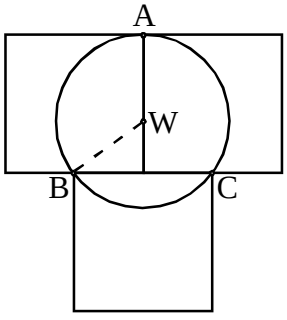


12. _____

13. A hexagon with 6cm sides has both a circumscribed circle and an inscribed circle, creating an annulus. Find the area of the entire annulus.

13. _____

14. Circle W passes through points A, B, and C on the congruent squares arranged below and has a radius of 15in. What is the side length of the squares?



14. _____